

الامتحان مكون من (5) أسئلة مكتوبة في صفحة واحدة و المطلوب الإجابة على كل الأسئلة

Time: 3 Hours

(1) Solve the following P.D.E:

(a) $3u_x + 4u_y + 10u = 5$ (b) $4u_{xx} - 5u_{xy} + u_{yy} + 2u_x - u_y - 8u = 0$

(2) Solve the LP problems:

(a) maximize $f = x + y + z - u$

s.t $x - y + z - u \leq 4$

$x + y - z + u \geq 6, \quad x, y, z, u \geq 0.$

(b) maximize $f = x + 2y$

s.t $x + y \leq 8$

$x + 4y = 20$

$3x + 2y \geq 6, \quad x, y \geq 0.$

(3)(a) Find the logarithmic curve that fits the data:

(1, 2), (2, 3), (3, 2.5), (4, 3.8), (5, 6)

(b) Find the table of differences of the following data and then find the value of y at $x = 2$: (1, 4), (3, 6), (5, 20), (7, 24).

(4)(a) Using the bisection method, number of iterations is 4, find a root to the equation: $f(x) = x^5 + 2x - 1 = 0$ in the interval $[0, 1]$

(b) Using the inverse interpolation, find a root to the equation:

$x^3 + 2x - 1 = 0$ in the interval $[0, 0.5]$

(5)(a) Show that the function $u(x, y) = x + \cos x \cdot \cosh y$ is harmonic and find its conjugate $v(x, y)$ such that the function $w = u + iv$ is analytic.

(b) Using Simpson's rule, evaluate the integral: $\int_1^{\infty} \frac{2}{1+x^2} dx$.

Good Luck

Dr. Mohamed H. Eid

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(1)(a) Solve the following LP problem, graphically:

$$\begin{aligned} &\text{maximize } f = 2x + y \\ &\text{s.t } \quad x + y \leq 8 \\ &\quad \quad x - y \leq 4 \\ &\quad \quad -x + y \leq 6, \quad x, y \geq 0. \end{aligned}$$

(b) Solve the LP problem:

$$\begin{aligned} &\text{maximize } f = x - 3y + 3z \\ &\text{s.t } \quad 2x + y - z \leq 9 \\ &\quad \quad x - y \leq 1 \\ &\quad \quad -3x + 2y + z \leq 6, \quad x, y, z \geq 0. \end{aligned}$$

(2) Solve the following P.D.E:

$$(a) u_x + u_y - 2u = \sqrt{2} \qquad (b) 6u_{xx} - 5u_{xy} + u_{yy} + u_x - u_y + 6u = 0$$

(3)(a) Find the logarithmic curve that fits the data:

$$(10, 1), (20, 3), (30, 4), (40, 7), (50, 8)$$

(b) Find the table of differences of the following data and then find the value of y at $x = 1.2$: $(1, 2), (2, 3), (3, 2), (4, 11), (5, 42)$.

(4)(a) Find the Lagrange polynomial $P_3(x)$ that satisfies the following data and find y, y' at $x = 1.5$: $(1, 1), (2, 3), (3, 13), (4, 37)$.

(b) Using the iterative method, number of iterations is 3, solve the system of equations:

$$\begin{bmatrix} 3 & 1 & -1 \\ 1 & 2 & -1 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix}$$

(5)(a) Find $u(x, y)$ and $v(x, y)$ of the function $f(z) = z \cos z$ and show that they satisfy Riemann's equations.

(b) Show that the function $u(x, y) = x^3 - 3xy^2 + y$ is harmonic and find its conjugate $v(x, y)$ such that the function $w = u + iv$ is analytic.

Good Luck

Dr. Mohamed H. Eid